Storage warehousing alternatives:

- Space can be rented in a public warehouse on the basis of storage volume for a specified period, without any capital investment for equipment.

- When there is a long-term need, a warehouse can be leased, and some investment for handling equipment might be required.

- A company whose main business is other than warehousing can operate a private warehouse as a separate division. Although, this alternative has several advantages over the others, owning a warehouse and handling equipment may result in substantial fixed and variable costs.

The storage capacity of a warehouse is defined as the amount of storage space required to accommodate the materials to be stored in order to meet a desired service level. Storage capacity depends rather significantly on the type of storage policy being used: dedicated/random...
DEDICATED STORAGE

Each product unit load is assigned to a fixed location based on throughput and storage space required.

The main advantage is the data handling efficiency, as each product has an predetermined address.

The products responsible for more of the travel activity between the warehouse and the docks should be assigned to locations closer to docks.

A measure of effectiveness is minimization of the total expected distance traveled.

\[ q = \# \text{ of storage locations} \]
\[ n = \# \text{ of products} \]
\[ m = \# \text{ of input/output points} \]
\[ s_j = \# \text{ of storage locations required by product } j \]
\[ t_j = \# \text{ of trips in/out of storage for product } j \]
\[ p_i = \% \text{ of travel in/out of storage to/from point } i \]
\[ d_{ik} = \text{ distance or time required to travel from point } i \text{ to location } k \]

\[ x_{ik} = \begin{cases} 
1 & \text{if the product is assigned to location } k \\
0 & \text{otherwise} 
\end{cases} \]
Minimize \( \sum_{j=1}^{n} \sum_{k=1}^{q} \frac{T_{jk}}{S_j} \sum_{i=1}^{m} p_i d_{ik} x_{jk} \)

s.t.,

\( \sum_{j=1}^{n} x_{jk} \leq 1 \quad k = 1, \ldots, q \)

\( \sum_{k=1}^{q} x_{jk} = S_j \quad j = 1, \ldots, n \)

\( x_{jk} \in \{0, 1\} \) for \( j = 1, \ldots, n \)

**Solution**

The objective function can be rewritten as:

\( \sum_{j=1}^{n} \frac{T_j}{S_j} \left( \sum_{k=1}^{q} \sum_{i=1}^{m} p_i d_{ik} x_{jk} \right) \)

let \( f_k = \sum_{i=1}^{m} p_i d_{ik} \); then

\( \sum_{j=1}^{n} \frac{T_j}{S_j} \left( \sum_{k=1}^{q} f_k x_{jk} \right) = \sum_{j=1}^{n} \frac{T_j}{S_j} \left( f_1 x_{j1} + f_2 x_{j2} + \cdots + f_q x_{jq} \right) \)

- Order the products according to their decreasing \( T_j/S_j \) values.
- Find the values of \( f_k \) (expected distance traveled between location \( k \) and the clock).
- Assign the locations in the warehouse to products according to their \( f \)-values from lowest to highest.
Example: 40 ft x 40 ft - dimensions of a rectangular warehouse

- One shipping & receiving dock located at the northeast corner.
- Two products A & B
- Total # of pallet per week for A is 100, for B is 80,
- Product A requires 10 storage bays, each 10x10 ft
- Product B requires 4 " " "

Solution:

\[ \frac{T_A}{S_A} = \frac{100}{10} = 10 \quad \frac{T_B}{S_B} = \frac{80}{4} = 20 \]

Order of products B, A.

Values of \( t_k \)

\[
\begin{array}{cccc}
10 & 20 & 20 & 10 \\
50 & 10 & 50 & 20 \\
50 & 50 & 40 & 30 \\
40 & 60 & 80 & 10 \\
\end{array}
\]

Final layout showing dedications!

RANDOM STORAGE

Incoming items are equally likely to be stored among all available storage spaces. In practice, incoming items are stored in the available location that is closest to the input/output point.
Under the assumption that the storage locations are highly utilized, indicated by a high rate of product moves into and out of storage locations, most actual warehouse operations can be modeled using a random storage policy. The main advantage of random storage is better utilization of the available storage space.

**Service Level Approach**

Minimize the amount of storage space subject to a specified probability of space shortage $\alpha (0 < \alpha < 1)$ is not exceeded.

$1 - \alpha$ is known as the service level.

A space shortage occurs when the available space capacity is insufficient to accommodate a storage-space requirement. It is assumed that the shortage is met using leased storage space.

Let $X_i$ be a random variable representing the inventory level of item $i$, $i = 1, \ldots, n$.

Let $X = \sum_{i=1}^{n} X_i$, assume that $X_i$ follows a uniform distribution.
\(X_i \sim N(a_i, b_i), \ i = 1 \ldots n\)

Assume that the items are labeled in non-decreasing order of \(b_i\).

\(S(\alpha)\) - The storage capacity at service level \(1-\alpha\),

\[P(X \leq S(\alpha)) \geq 1-\alpha.\]

Cost-Based Approach

The total storage space is found by minimizing the sum of costs associated with owned storage space and contracted storage space for accommodating space shortages regardless of service level.

Total storage cost usually includes a fixed and a variable component.

The fixed cost is incurred when purchasing material handling equipment. The variable cost is given per storage area unit.

\(y\) - owned warehouse area

\(C(y)\) - cost of owned warehouse area associated with \(y\)
A piecewise linear cost function is assumed.

\[ C_i^1(y) = f_i + v_i Y \quad 0_i \leq y \leq 0_{i+1} \]

where \( f_i \) - fixed cost for the \( i \)th interval of \( y \).

\( v_i \) - variable cost per unit of owned warehouse area for the \( i \)th interval of \( y \).

\( Y \) - leased warehouse area

\[ C^2(y) = F_j + v_j Y \quad L_j \leq Y \leq L_{j+1} \]

where \( F_j \) - fixed cost for \( j \)th interval of \( Y \).

\( v_j \) - cost per unit area of the variable leased warehouse area for the \( j \)th interval of \( Y \).

In general, \( 0_1 \) & \( L_1 \) is set to be zero.