S/R Machine Cycle Times:

To determine the time required to perform an operation (storage or retreival), expected S/R machine cycle time formulas will be used.

The S/R machine is capable of traveling simultaneously in the aisle both vertically and horizontally. Hence, the time required to travel from the P/D station to a storage or retreival location is the maximum of the horizontal and vertical travel times, which is also known as Chebyshev travel.

A single command cycle consists of either a storage or a retreival, but not both, whereas a dual command cycle involves both a storage and a retreival.

- A single command storage cycle begins with the S/R at the P/D station; it picks up a load, travels to the storage location, deposits the load, and returns empty to the P/D station.
- A single command retreival cycle begins with the S/R at the P/D station; it travels empty to the retreival location, picks up the load, travels to the P/D station and deposits the load.

One pickup and one deposit occur in each single command cycle.

· A dual command cycle begins with the S/R at the P/D station; it picks up a load, travels to the storage location, deposits the load, travels empty to the retreival location, picks up the load, travels to the P/D station, and deposits the load.

A total of two pickups and two deposits are performed during a dual command cycle.

E(SC) = the expected travel time for a single command cycle

E[TB] = the expected travel time from the storage location to the retreival location during a dual command cycle.

E[DC] = the expected travel time for a dual command cyc

L = the rack length in feet.

H = the rack height in feet.

h_v = the horizontal velocity of the S/R machine (ft/min)

V_v = the vertical velocity of the S/R machine (ft/min)

t_h = time required to travel horizontally from the P/D station to the furthest location in the aisle.

$t_h = L / h_v$.

t_v = time required to travel vertically from the P/D station to the furthest location in the aisle. $t_v = H / v_v$.

Normalize; by dividing its shorter side by its longer side.

$T = \max(t_h, t_v)$ "scaling factor"

$Q = \min(t_h/T, t_v/T)$ "shape factor"

$$E[SC] = T\left[1 + \frac{Q^2}{3}\right]$$

$$E[TB] = \frac{T}{30}\left[10 + 5Q^2 - Q^3\right]$$

$$E[DC] = \frac{T}{30}\left[40 + 15Q^2 - Q^3\right]$$

$$T_{SC} = E[SC] + 2\,T_{P/D}$$

$$T_{DC} = E[DC] + 4\,T_{P/D}$$

$$E[z] = \int_0^b z f(z)\,dz + \int_b^1 z f(z)\,dz \qquad \underline{\underline{\max(x,y)}}$$

$$f(z) = \begin{cases} z^2/b & 0 \le z \le b \\ z & b \le z \le 1 \end{cases}$$

$$f(z) = \begin{cases} 2z/b & 0 \le z \le b \\ 1 & b \le z \le 1 \end{cases}$$

$$= \int_0^b z \cdot \frac{2z}{b}\,dz + \int_b^1 z \cdot 1 \cdot dz$$

$$= \left[\frac{2z^3}{3b}\right]_0^b + \left[\frac{z^2}{2}\right]_b^1$$

$$= \left(\frac{2b^{\not{3}2}}{3\not{b}} - 0\right) + \left[\frac{1}{2} - \frac{b^2}{2}\right]$$

$$= \left(\frac{2}{3}b^2 - \frac{b^2}{2} + \frac{1}{2}\right)$$

$$= \left(\frac{(4-3)}{6}b^2 + \frac{1}{2}\right)$$

$$E[z] = \left(\frac{b^2}{6} + \frac{1}{2}\right)$$

Example:

Suppose AS/RS is to be designed for 40" x 48" (depth x width) unit loads that are 48" tall. There are to be eight aisles; each aisle is 12 loads high and 80 loads long.

Assume the S/R requires 0.35 min to perform a P/D operation, travels horizontally at an average speed of 350 fpm, and travels vertically at an average speed of 60 fpm.

The dimensions (inches) of the storage rack are 56m (length) by 58n (height), where m is the # of horizontal addresses, and n is the # of vertical addresses.

m = 80 and n = 12, therefore;

$$L = 56 \times (80)/12 = 373.33 \text{ ft}$$

$$H = 58 \times (12)/12 = 58.00 \text{ ft}$$

The maximum horizontal and vertical travel times are determined as follows:

$$t_h = \frac{L}{h_v} = \frac{373.33 \text{ ft}}{350.00 \text{ ft/min}} = 1.067 \text{ min}$$

$$t_v = \frac{H}{V_v} = \frac{58.00}{60.00} = 0.967 \text{ min}$$

Thus; $T = \max(1.067, 0.967) = 1.067$ min and

$$Q = \frac{0.967}{1.067} = 0.906$$

Therefore, the sigle commod and dual commod cycle times are;

$$T_{SC} = T\left(1 + \frac{Q^2}{3}\right) + 2\,T_{P/D}$$

$$= 1.067\left(1 + \frac{(0.906)^2}{3}\right) + 2 \times (0.35) = 2.06 \text{ min}$$

per sigle commod cycle!

$$T_{DC} = \frac{T}{30}\left(40 + 15Q^2 - Q^3\right) + 4\,T_{P/D}$$

$$= \frac{1.067}{30}\left(40 + 15(0.906)^2 - (0.906)^3\right) + 4 * (0.35)$$

$= 3.23$ min per dual command cycle

Thus, the average cyle time per operation is;

2.06 min with a single commond cycle, and

1.615 min with a dual commod cycle.

Suppose, 40% of the storages and 40% of the retreivals are performed as single command operations; the remaining are performed as dual command operations.

If 120 storages per hour and 120 retreivals per hour are to be handled by the AS/RS; assuming the 8 S/R machines are loaded uniformly; then each S/R must perform 15 storages per hour and 15 retreivals per hour. Since, 6 storages per hour and 6 retreivals per hour are performed using a single command operation, there will be 9 dual command trips per hour.

The workload on the S/R machine is:

$$2 * 6 * 2.06 + 9(3.23) = 53.79 \text{ min/hour}$$

$$\% \text{ utilization} = \frac{53.79}{60.00} = 89.65\%$$

$$\text{Average cycle time per operation} = \frac{53.79}{30} = 1.793 \text{ min}$$

that implies that the maximum throughput capacity of the system is equal to $(60/1.793) * 8 = 267.71$ operations/hour.

Since the times between requests for storage and/or retreivals are often random variables and since the locations of the storage and/or retreival addresses to be visited on a single or dual command cycle are random variables waiting line analysis (Queueing) or simulation can be used to aid in the AS/RS design.