Travel-Time Models for Automated Storage/Retrieval Systems*

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Abstract: Travel-time models are developed for automated storage/retrieval (AS/R) machines. The S/R machine travels simultaneously horizontally and vertically as it moves along a storage aisle. For randomized storage conditions expected travel times are determined for both single and dual command cycles. Alternative input/output (I/O) locations are considered. Additionally, various dwell-point strategies for the storage/retrieval machine are examined.

■ Automated storage/retrieval (AS/R) systems are of strong current interest due to such benefits as lower building and land cost, labor savings, reduced inventory levels, and an improved throughput level, among others [19]. In this paper, analytical expressions are derived for travel times for the storage machine in an AS/RS. A variety of storage rack shapes, I/O locations, and dwell-point strategies are examined. Both single and dual command systems are included. The principal assumptions for the analysis are: a continuous approximation to the discrete rack face, randomized storage assignment, and Tchebychev travel for the S/R. (Note that this is simultaneous horizontal and vertical travel in contrast to rectilinear or sequential travel, as performed by a lift truck.)

The travel-time expressions developed extend the results of Graves, Hausman, and Schwarz (GHS) for randomized storage in their seminal papers on the subject [6, 9, 15]. GHS considered a variety of storage methods (random, dedicated, and class-based); they assumed that the I/O point is located at the corner of the rack and that the rack is square in time. Alternative I/O locations and rack configurations are considered here.

The travel-time expressions presented can be used to establish throughput standards on existing systems. The results are also quite useful in estimating throughput performance for first cut evaluations of AS/RS design configurations.

LITERATURE SURVEY

Much of the previous work on AS/R systems is based on simulation [1, 2, 12, 14, and 15]. Bafna [1] developed a design package where the optimum configuration is determined by using simulation in conjunction with a search procedure. A similar approach is presented by Koenig [12], where the search for the optimum configuration is limited to certain values of the design variables specified by the user. Other simulation studies have addressed the throughput performance of AS/R systems under different operating and/or storage policies assuming that the design (configuration) of the system is known [2, 14, and 15].

A design package based on analytical techniques was developed by Bozer and White [4]. Based on Zollinger's [18] cost model, the mathematical properties of the cost

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functions were defined corresponding to various elements in the system. Subsequently, the minimum-cost design was determined by performing a Fibonacci search over the number of aisles in the system.

Karasawa, Nakayama, and Dohi [11] developed a cost model of an AS/RS. They considered only single command cycles. Their total cost function included terms for storage racks, S/R machines, building, and land. The resulting non-linear programming problem was solved by using Lagrangian multipliers and then choosing the best neighborhood integer solution.

A study comparing randomized storage with turn-over based, i.e., dedicated storage, is presented by Hausman, Schwarz, and Graves in [9]. The basis of comparison is the expected S/R machine travel time. The rack is assumed to be square in time and no interleaving, i.e., dual command cycles are allowed. Using a continuous approximation to represent the rack, the expected one-way travel time under random storage assignment is found to be 2/3 time units for a square rack with unit travel time in the horizontal and vertical directions. Representing the ABC curve as a continuous turnover function, they show the percentage improvement gained in expected S/R machine travel by using full turnover-based storage instead of randomized storage. Likewise, they show the percentage improvement gained by using class-based storage for two-class and three-class storage assignments. Lastly, based on empirical results, they determine that under randomized storage the continuous approximation generates satisfactory results relative to the discrete rack. However, for turnover-based storage assignments, the continuous approximation is shown to underestimate the actual travel time as the skewness of the ABC curve is increased.

Another paper presented by Graves, Hausman, and Schwarz [6] is an extension to their study presented in [9]. The rack is assumed to be square in time, with the I/O point located at one corner of the rack. In [6] Graves, Hausman, and Schwarz present analytical and empirical results for various combinations of alternative storage assignment rules and scheduling policies. Each alternative is compared on the basis of expected S/R machine travel time. Their results indicate that, under the assumptions outlined in their study, a reduction in expected S/R machine travel time can be obtained by using turnover-based storage instead of randomized storage. The largest reduction is generated by using full turnover-based storage, which is followed by three-class and two-class turnover-based storage assignments.

Hodgson and Lowe [10] studied a layout problem involving the placement of items in a storage rack serviced by an S/R machine. Their analysis was restricted to the case of dedicated storage and single command cycles.

Previous studies have addressed the case of rectilinear or sequential travel of storage/retrieval machines. Mayer [13] appears to have performed one of the earliest studies of travel times for rectilinear travel; he studied manual order picking, the use of order picker trucks, and the use of lift trucks for both single and dual command cycles.

It appears that Gudehus [7] independently obtained travel time results similar to ours for racks that are not square in time. His approach appears to be similar to that of Graves, Hausman, and Schwarz.

EXISTING METHODS

As noted previously, analytical approaches have been developed for determining the expected travel time for an AS/RS. Graves, Hausman, and Schwarz in [6] and [9] assumed the rack to be square in time. That is, the dimensions of the rack and the vertical and horizontal speeds of the S/R machine are such that the time to reach the row most distant from the I/O point equals the time to reach the most distant column, given that the I/O point is located at the lower left-hand corner of the rack.

Empirical experience indicates that the optimum design for AS/R systems frequently is not square in time. Furthermore, many systems that have been installed are not square in time. For this reason it is desirable to determine the expected travel time for a rack that is not necessarily square in time.

An alternate method of estimating the expected travel time is given by the AS/RS Product Section of The Material Handling Institute, Inc. (MHI) [19]. The single-command expected travel time is estimated to be equal to twice the time required to travel from the I/O point to the storage slot at the center of the rack. In order to determine the expected round-trip travel time for dual command cycles, it is assumed that a storage is performed at the center of the rack and a retrieval is performed at a point located three-fourths of the distance (horizontally and vertically) from the I/O point. The MHI travel-time model may not provide an accurate representation of the travel of the S/R machine. An empirical evaluation of the MHI model will be presented later.

To facilitate the determination of the travel time for an S/R machine, the following notation is introduced:

N is the total number of openings in the rack.

t_{0i} is the one-way travel time between the I/O point and the ith opening ($t_{0i} = t_{i0}$).

t_{ij} is the one-way travel time between the ith opening and the jth opening ($t_{ij} = t_{ji}$).

$E(\overline{SC})$ is the expected single-command round-trip travel time.

$E(\overline{DC})$ is the expected dual-command travel time.

The expected single-command travel time can be computed from the following expression:

$$E(\overline{SC}) = \frac{1}{N} \sum_{i=1}^{N} 2t_{0i} . \quad (1)$$

The expected dual command travel time is given by

$$E(\overline{DC}) = \frac{2}{N(N-1)} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} [t_{0i} + t_{ij} + t_{j0}] . \quad (2)$$

A STATISTICAL APPROACH

Closed-form expressions and the methodology used to develop the travel times are now presented. The following assumptions are made:

- The rack is considered to be a continuous rectangular pick face where the I/O point is located at the lower left-hand corner.
- The S/R machine operates either on a single or dual command basis, i.e., multiple stops in the aisle are not allowed.
- The rack length and height, as well as the S/R machine speed in the horizontal and vertical directions, are known.
- The S/R machine travels simultaneously in the horizontal and vertical directions. In calculating the travel time, constant velocities are used for horizontal and vertical travel.
- Randomized storage is used. That is, any point within the pick face is equally likely to be selected for storage or retrieval.
- Pick-up and deposit (P/D) times associated with load handling are ignored. The P/D time is generally independent of the rack shape and the travel velocity of the S/R machine. Furthermore, given the load characteristics, the P/D time is usually deterministic. Hence, it is a straightforward matter to include the P/D time after the average travel time has been computed.

As stated earlier, the pick face is a continuous rectangle with known dimensions that can vary from one application to another. To standardize the pick face, use the definitions:

s_h is the speed of the S/R machine in the horizontal direction.

s_v is the speed of the S/R machine in the vertical direction.

L is the length of the rack.

H is the height of the rack.

Now let t_h represent the horizontal travel time required to go to the farthest column from the I/O station. Likewise, let t_v denote the vertical travel time required to go to the farthest row (level). Then by definition $t_h = L/s_h$ and $t_v = H/s_v$. Let $T = \text{Max}\{t_h, t_v\}$ and $b = \text{Min}\{t_h/T, t_v/T\}$, which implies that $0 \leq b \leq 1$. In subsequent discussions, b is referred to as the "shape factor." Note that if the rack dimensions and travel velocities are such that $t_h = t_v$, then $b = 1$ and the rack is said to be square in time. Without loss of generality, assume that $T = t_h$; that is, $b = t_v/T$.

We will consider first the single-command travel time. Let the storage (or retrieval) point be represented by (x,y) in time, where $0 \leq x \leq 1$ and $0 \leq y \leq b$. Travel from (0,0) to (x, y), say t_{xy}, will be $t_{xy} = \text{Max}(x, y)$. Now let $G(z)$ denote the probability that travel time to (x, y) is less than or equal to z. Assuming the x, y coordinates are independently generated,

$$G(z) = \Pr(t_{xy} \leq z) = \Pr(x \leq z)\Pr(y \leq z) .$$

Furthermore, for randomized storage the coordinate locations are assumed to be uniformly distributed. Thus,

$$\Pr(x \leq z) = z$$

and

$$\Pr(y \leq z) = \begin{cases} z/b & \text{if} \quad 0 < z \leq b , \\ 1 & \text{if} \quad b \leq z \leq 1 . \end{cases}$$

Hence

$$G(z) = \begin{cases} z^2/b & \text{for } 0 \leq z \leq b \\ z & \text{for } b < z \leq 1 . \end{cases}$$

Therefore the probability density function, $g(z)$, will be

$$g(z) = \begin{cases} 2z/b & \text{for } 0 \leq z \leq b \\ 1 & \text{for } b < z \leq 1 . \end{cases}$$

Letting $E(SC)$ denote the expected travel time under single command for the normalized rack, the following is obtained

$$E(SC) = 2 \int_{z=0}^{1} z\, g(z) dz = \frac{1}{3} b^2 + 1 . \quad (3)$$

(Note: for $b = 1$, $E(SC) = 4/3$, as obtained by GHS.)

Next, consider the dual command cycle. By definition, each dual command cycle involves two random locations; one representing the storage point, the other representing the retrieval point. To analyze the expected travel time between the two points, recall that any point is represented as (x, y) in time and $0 \leq x \leq 1$ and $0 \leq y \leq b$. Let t_B be the time required to travel between the storage and retrieval locations. Also, let

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$$F(z) = \Pr(t_B \leq z) = \Pr(|x_1 - x_2| \leq z)\Pr(|y_1 - y_2| \leq z),$$

where (x_1, y_1) and (x_2, y_2) are the two random points. First consider the term $\Pr(|y_1 - y_2| \leq z)$. Recall, if $y_{(1)}, \ldots, y_{(n)}$ are the order statistics of a sample $y_1, \ldots, y_n$, from a population with probability distribution function $f(y)$ and cumulative distribution function $F(y)$, the difference $y_{(n)} - y_{(1)}$ is called the sample range R. Letting $H(r) = \Pr(R \leq r)$, in [8] it is shown that

$$H(r) = n \int_{v=-\infty}^{\infty} f(v)[F(v+r) - F(v)]^{n-1} dv. \quad (4)$$

Differentiating $H(r)$ yields the probability density function

$$h(r) = n(n-1) \int_{v=-\infty}^{\infty} [F(v+r) - F(v)]^{n-2} f(v) f(v+r)\, dv. \quad (5)$$

Recall that $0 \leq y_1 \leq b$ and $0 \leq y_2 \leq b$. Hence

$$f(y) = \begin{cases} 1/b & 0 \leq y \leq b, \\ 0 & \text{otherwise} \end{cases} \text{for } F(y) = \begin{cases} 0 & y < 0 \\ y/b & 0 \leq y \leq b. \\ 1 & y > b. \end{cases}$$

It should be noted that $f(y)$ and $F(y)$ are derived based on the assumption that under randomized storage, y_1 and y_2 are uniformly distributed in the interval (0, b). Since $0 \leq R \leq b$, then $0 \leq v \leq b - r$. Letting $n = 2$, from Equation (5)

$$h(r) = 2 \int_{v=0}^{b-r} f(v) f(v+r) dv = \frac{2}{b^2}(b - r). \quad (6)$$

Letting

$$F_y(z) = \Pr(|y_1 - y_2| \leq z),$$

then

$$F_y(z) = \Pr(0 \leq R \leq z) = \frac{2}{b^2} \int_0^z (b - r) dr.$$

Therefore

$$F_y(z) = \begin{cases} 2z/b - z^2/b^2 & \text{for } 0 \leq z \leq b, \\ 1 & \text{for } b < z \leq 1 \end{cases} \quad (7)$$

Next consider the term $\Pr(|x_1 - x_2| \leq z)$, where $0 \leq x_i \leq 1$ for $i = 1, 2$. Let $F_x(z) = \Pr(|x_1 - x_2| \leq z)$. Letting $b = 1$ in Equation (7)

$$F_x(z) = 2z - z^2 \text{ for } 0 \leq z \leq 1. \quad (8)$$

Recall that $F(z) = \Pr(t_B \leq z)$ where t_B represents the travel between time. Based on Equations (7) and (8), $F(z)$ can be written as $F(z) = F_x(z) F_y(z)$ or,

$$F(z) = \begin{cases} (2z - z^2)(2z/b - z^2/b^2) & \text{for } 0 \leq z \leq b \\ (2z - z^2) & \text{for } b < z \leq 1. \end{cases}$$

Therefore, the probability density function is

$$f(z) = \begin{cases} (2-2z)(2z/b - z^2/b^2) + (2z - z^2)(2/b - 2z/b^2) & \text{if } 0 \leq z \leq b, \\ 2 - 2z & \text{if } b < z \leq 1. \end{cases}$$

Letting $E(TB)$ denote the expected travel time between the two randomly selected points, the following result is obtained:

$$E(TB) = \int_0^1 z f(z) dz = \frac{1}{3} + \frac{1}{6} b^2 - \frac{1}{30} b^3. \quad (9)$$

(Note: for $b = 1$, $E(TB) = 7/15$, as obtained by GHS).

Let $E(DC)$ denote the expected travel time for a complete dual command cycle with a normalized rack. Then, by definition, $E(DC) = E(SC) + E(TB)$. Hence

$$E(DC) = \frac{4}{3} + \frac{1}{2} b^2 - \frac{1}{30} b^3. \quad (10)$$

Using Equations (3) and (10), the expected single-command and dual-command travel times can be calculated for a normalized rack.

Example: Suppose that rack dimensions and the S/R machine speed is such that $L = 348$ ft, $H = 88$ ft, $s_h = 356$ fpm, and $s_v = 100$ fpm. Using the approach developed earlier, we have $t_h = L/s_h = 348/356 = 0.9775$ min and $t_v = H/s_v = 88/100 = 0.8800$ min. Therefore, $T = 0.9775$ and $b = 0.90$. Hence, the normalized rack is 0.900 time units long in the vertical direction and 1.0 time units long in the horizontal direction. Using Equations (3) and (10), we obtain $E(SC) = 1.27$ time units and, $E(DC) = 1.7140$ time units. To obtain the results corresponding to the original rack, we denormalize the above travel times to obtain $E(SC) = E(SC)T = 1.2414$ min and $E(\overline{DC}) = E(DC)T = 1.6754$ min.

COMPARISON WITH OTHER METHODS

The continuous model is now empirically evaluated for accuracy. More specifically, the results obtained from the model are compared with those obtained from a discrete rack. The expected travel time for the discrete rack is obtained from a computer code using Equations (1) and (2) for calculating $E(\overline{SC})$ and $E(\overline{DC})$, respectively. The analysis also includes those cycle times obtained from the MHI model.

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Table 1: Comparison of single command travel times.

Alternative 1								
No. of levels	10	9	8	7	6	5	4	3
No. of columns	40	45	50	57	67	80	100	133
NOP	400	405	400	399	402	400	400	399
Shape factor	1.0000	0.8000	0.6400	0.4912	0.3582	0.2500	0.1600	0.0902
Discrete rack	0.5330	0.5457	0.5680	0.6156	0.6985	0.8165	1.0084	1.3335
Cont. model	0.5333	0.5460	0.5683	0.6158	0.6987	0.8167	1.0085	1.3336
(% deviation)	0.0625	0.0543	0.0469	0.0380	0.0285	0.0204	0.0132	0.0075
MHI model	0.4000	0.4600	0.5000	0.5800	0.6800	0.8000	1.0000	1.3400
(% deviation)	25.000	15.700	11.970	5.780	2.650	2.020	0.830	0.490
Alternative 2								
No. of levels	5	X	4	X	3	X	2	X
No. of columns	20		25		33		50	
NOP	100		100		99		100	
Shape factor	1.0000		0.6400		0.3636		0.1600	
Discrete rack	0.2660		0.2836		0.3441		0.5040	
Cont. model	0.2667		0.2841		0.3445		0.5043	
(% deviation)	0.2506		0.1881		0.1174		0.0529	
MHI model	0.2400		0.2600		0.3400		0.5000	
(% deviation)	9.7700		8.3200		1.1900		0.7900	

Two alternative configurations were developed for the purpose of the above evaluation. The first configuration has a rack length of 40 columns and a rack height of 10 levels, which yields 400 openings. The second configuration, representing a smaller system, has 20 columns and 5 levels, with 100 openings. For both alternatives, it is assumed that the width and the height of each rack opening is 4 ft and that the S/R machine travels at 400 fpm and 100 fpm in the horizontal and vertical directions respectively. Subsequently, the number of levels and columns in each rack is varied to obtain different shape factors while keeping the number of openings (NOP) approximately equal.

The results obtained for the single command cycle are presented in Table 1. It can be observed that the continuous model displays a satisfactory performance with the largest percentage deviation being 0.2506%. It is also observed that the performance of the continuous model improves as the rack becomes nonsquare. The same observation can be made for the cycle times obtained from the MHI model. In fact, the MHI model results improve significantly as the shape factor goes to zero. This can be attributed to the fact that, as the discrete rack becomes nonsquare, the total number of openings associated with a specific travel time approaches one. Hence, the expected single command travel time approaches that value obtained by using the midpoint of the rack as the storage (or retrieval) point. Note that the MHI model significantly underestimates the travel time, especially for those configurations with large shape factors. The maximum deviation observed is 25%, almost 100 times the maximum deviation observed under the continuous model!

The results obtained for the dual command cycle are presented in Table 2. As before, the continuous model performs in a satisfactory manner with the largest percentage deviation being 0.2069%. Based on the results given in Table 2, the continuous model appears to perform best when $b = 0.35$. This phenomena can be explained as follows: first, recall that $E(DC) = E(SC) + E(TB)$. Based on Table 1, $E(SC)$ is consistently overestimated. Using $E(DC)$ given in Table 2 and $E(SC)$ given in Table 1, one can obtain the values for $E(TB)$. In doing so, one will observe that $E(TB)$ is overestimated for $b = 1$ and underestimated for $b = 0$. Hence, at certain b values the overestimation in $E(SC)$ will be offset by the underestimation in $E(TB)$, yielding an accurate $E(DC)$ value.

Similar observations can be made for the results obtained from the MHI model in Table 2. However, the best b value of the MHI model appears to be $b = 0.64$. The difference is due to the fact that the MHI approach involves a different method for computing $E(TB)$. It should be noted that the maximum percentage deviation observed for the MHI model is 16.62%, almost 80 times the maximum deviation observed under the continuous model.

Lastly, based on Tables 1 and 2, it can be observed that the performance of the continuous model improves as the number of openings are increased. Such a result is intuitive since the continuous model ignores the discreteness of the rack; the impact of doing so reduces as one increases the number of openings associated with a given shape factor.

From Equations (3 and 10), it can be shown that a square-in-time rack minimizes travel time. To do so for a single command cycle, minimize $T(b^2/3 + 1)$ subject to

Table 2: Comparison of dual command travel times.								
Alternative 1								
No. of levels	10	9	8	7	6	5	4	3
No. of columns	40	45	50	57	67	80	100	133
NOP	400	405	400	399	402	400	400	399
Shape factor	1.0000	0.8000	0.6400	0.4912	0.3582	0.2500	0.1600	0.0902
Discrete rack	0.7196	0.7360	0.7644	0.8264	0.9353	1.0914	1.3464	1.7795
Cont. model	0.7200	0.7363	0.7647	0.8265	0.9353	1.0912	1.3460	1.7787
(% deviation)	0.0511	0.0430	0.0325	0.0177	0.0000	0.0162	0.0318	0.0449
MHI model	0.6000	0.6800	0.7600	0.8600	1.0200	1.2000	1.5000	2.0000
(% deviation)	16.620	7.6100	0.5800	4.0700	9.0600	9.9500	11.410	12.390
Alternative 2								
No. of levels	5		4		3		2	
No. of columns	20		25		33		50	
NOP	100		100		99		100	
Shape factor	1.0000		0.6400		0.3636		0.1600	
Discrete rack	0.3593		0.3818		0.4613		0.6739	
Cont. model	0.3600		0.3823		0.4613		0.6730	
(% deviation)	0.2069		0.1315		0.0000		0.1277	
MHI model	0.3200		0.3800		0.5000		0.7600	
(% deviation)	10.940		0.4700		8.3900		12.780	

$bT^4 = A$, where A is the required storage area. Substituting $\sqrt{A/b}$ for T, taking the derivative of the denormalized cycle time with respect to b, setting the result equal to zero, and solving for b yields a value of $b = 1.0$. The solution is easily shown to be both necessary and sufficient. A similar approach can be used for dual command cycles.

DWELL POINT STRATEGIES

The dwell point is referred to as the location of the S/R machine when it becomes idle. The following are representative of the dwell point strategies used for locating the S/R machine following completion of storage and retrieval operations:

A Return to the input station following the completion of a single command storage; remain at the output station following the completion of either a single command retrieval or a dual command cycle.

B Remain at the storage location following the completion of a single command storage; remain at the output station following the completion of either a single command retrieval or a dual command cycle.

C Travel to a midpoint location in the rack following the completion of any cycle.

D Travel to the input station following the completion of any cycle.

The appropriate dwell point strategy depends on the timing of the storages and retrievals as suggested in the following section.

ALTERNATIVE CONFIGURATIONS FOR THE I/O POINT

In the previous discussion, it was assumed that the I/O point is located at the lower left-hand corner of the rack and every trip originates and terminates at the I/O point. The assumption is now relaxed; four alternative configurations are analyzed; and the corresponding expected travel time expressions are developed.

Input and Output at Opposite Ends of the Aisle

For the first configuration to be considered, assume that all storage orders are initiated at the input station while all retrieval orders are terminated at the output station. Dwell point strategy A is assumed. Consequently, at the time the S/R machine starts a dual cycle, it will be at the input station if the previous trip was a storage; otherwise it will be at the output station. The expected travel time from any corner of the rack to a randomly selected point or vice versa, say $E(V)$, can be obtained by dividing $E(SC)$ by 2. Also, recall the expected travel time between two randomly selected points, $E(TB)$, is given by Equation (9). Note that $E(TB)$ will remain constant regardless of the locations of the input and output stations.

Suppose there are N operations to be performed per hour, where an operation is defined to be either a storage or a retrieval. Furthermore, suppose operations are equally storages and retrievals, such that $N/2$ storages are to be performed per hour. If α percent of the storages are performed using single command cycles, then $(1 - \alpha)$ percent of the storages will be performed using dual command cycles. Recall, a dual command cycle involves both a storage and a retrieval. Thus, the average number of dual command cycles to be performed per hour will be $(1 - \alpha)N/2$. The average number of single command cycles per hour will be the sum of the single command storages and retrievals, or αN.

Based on the statistical arguments given above, the probability of a single command storage is $\alpha/2$. Likewise, the probability of either a single command retrieval or a dual command is $1 - (\alpha/2)$. Thus, a trip will terminate at the input station with probability $\alpha/2$. Likewise, with probability $1 - (\alpha/2)$, the trip will terminate at the output station.

If the S/R machine is at the input station, then the expected travel time to perform a storage is equal to $2E(V)$, a retrieval is equal to $2E(V)$, and a dual trip is equal to $2E(V) + E(TB)$.

If the S/R machine is at the output station, then the expected travel time to perform a storage is equal to $2E(V) + K$, a retrieval is equal to $2E(V)$, and a dual trip is equal to $2E(V) + E(TB) + K$, where K is the fixed travel time from the output to the input station.

Defining an operation as a storage or a retrieval, the expected travel time per operation, say $E_1(T)$, can be shown to be

$$E_1(T) = E(V)(1+\alpha) + \frac{1}{2}E(TB)(1-\alpha) + \frac{1}{2}K\left(1 - \frac{\alpha}{2}\right). \quad (11)$$

Suppose the S/R machine is not required to return to the input station after a storage. Instead, assume dwell point strategy B applies. The S/R machine remains at the point of storage, awaiting the next order. If the next order is a storage or dual command order, the S/R machine returns to the input station. Otherwise, it travels directly to the retrieval point. Hence, a trip will terminate at some point within the rack with probability $\alpha/2$. Likewise, with probability $1 - (\alpha/2)$ the trip will terminate at the output station. Note that under dwell point strategy B, a trip will never terminate at the input station.

At the start of a cycle, if the S/R machine is at the output station, then the expected travel time to perform a storage is equal to $E(V) + K$, a retrieval is equal to $2E(V)$, and a dual trip is equal to $2E(V) + E(TB) + K$. If the S/R machine is within the rack, then the expected travel time to perform a storage is equal to $2E(V)$, a retrieval is equal to $E(TB) + E(V)$, and a dual trip is equal to $3E(V) + E(TB)$. Hence, the expected travel time per operation, $E_1(T)$, can be shown to be

$$E_1(T) = (1-\alpha/2)\left\{\frac{1}{2}E(V)(\alpha+2) + \frac{1}{2}E(TB)(1-\alpha) + \frac{1}{2}K\right\} + \frac{\alpha}{2}\left\{\frac{3}{2}E(V) + \frac{1}{2}E(TB)\right\}. \quad (12)$$

Finally, it is worthwhile to compare the two dwell point strategies on an expected travel time basis. The second strategy is expected to perform better, for under the second strategy the S/R machine remains within the rack and travels to the input station only if the next order is a storage. Assuming $\alpha = 1$, from Equation (12)

$$E_1(T) = \frac{3}{2}E(V) + \frac{1}{4}K + \frac{1}{4}E(TB). \quad (13)$$

Hence, from Equations (11 and 13), the following ratio, ϕ, is obtained.

$$\phi = \frac{E_1(T)\text{ under strategy B}}{E_1(T)\text{ under strategy A}} = \frac{\frac{3}{2}E(V) + \frac{1}{4}K + \frac{1}{4}E(TB)}{2E(V) + \frac{1}{4}K} \quad (14)$$

Assuming $b = 1$, $E(V) = 2/3$, and $E(TB) = 7/15$. Furthermore, for a normalized rack, $K = 1$. Substituting these values into Equation (14) gives $\phi = 0.86$. Hence, for $b = 1$, using the second dwell point strategy generates a 14% reduction in the expected travel time for a single command trip.

Input and Output at the Same End of the Aisle, But at Different Elevations

The second configuration to be considered is assumed to have the input station at the lower left-hand corner of the rack while the output station is located d time units above the input station, where $d < b$. Furthermore, it is assumed that the vertical travel yields the value of b.

Consider the single command cycle first. As shown in Fig. 1, the rack can be visualized as being two separate racks (as indicated by the dashed line). Travel from the input station to a random point in the rack can be expressed as

$$E(V) = \frac{1}{6}b^2 + \frac{1}{2}. \quad (15)$$

However, on going from the random point to the output station, Equation (15) is not appropriate. The output station may be considered to be located at the corner of racks A and B. Due to symmetry, Equation (15) holds as long as the station is located at one of the corners. Hence, denoting the average return time from Section A as $E_A(V)$ and using Equation (15) gives

A 0007

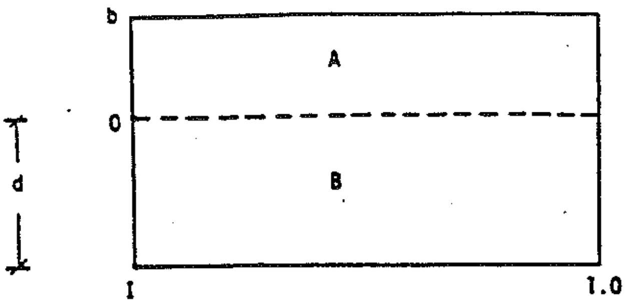

Fig. 1. Input and output at the same end of the aisle, but at different elevations.

$$E_A(V) = \frac{b^2 - 2bd + d^2}{6} + \frac{1}{2}. \qquad (16)$$

Similarly,

$$E_B(V) = \frac{d^2}{6} + \frac{1}{2} \qquad (17)$$

Now, Let $E_0(V)$ denote the expected travel time for returning to the output station. Thus,

$$E_0(V) = p_1 [E_A(V)] + (1 - p_1)[E_B(V)], \qquad (18)$$

where p_1 is the probability that the return trip originates from section A. Due to randomized storage, p_1 = area of section A/total rack area, i.e., $p_1 = (b - d)/b$. Hence, Equation (18) can be given as

$$E_0(V) = \frac{1}{6}b^2 + \frac{1}{2} - \frac{1}{2}d(b - d). \qquad (19)$$

Next consider the dual command cycle. By definition, travel time between two randomly selected points in the rack is independent of the location of the input and output stations. Hence, the expression developed earlier for $E(TB)$ is still valid.

Assume that the system operates according to dwell point strategy A defined previously. Using an approach similar to that one used for the first configuration, the expected travel time per operation, $E_2(T)$, can be shown to be

$$E_2(T) = \frac{\alpha}{2}\left\{\alpha E(V) - \frac{1}{2}\alpha E(TB) + \frac{1}{2}[E(V) + E(TB) + E_0(V)]\right\}$$
$$+ (1 - \alpha/2)\left\{\frac{1}{2}\alpha[E(V) - E(TB) + E_0(V)] + \frac{1}{2}[E(V) + E(TB) + E_0(V)] + \frac{1}{2}d\right\}. \qquad (20)$$

It is also worthwhile to compare the results obtained for the second configuration with those obtained for the first, assuming that the same dwell point strategy is used for both configurations. Recall, the expected travel time per operation for the first configuration, $E_1(T)$, is given by Equation (11). For the purposes of the comparison let α=0.50, d = 0.50b, and b = 1. By definition, K = 1. The values of $E(V)$, $E(TB)$ and $E_0(V)$ can be determined by using Equations (15), (9), and (19) respectively. Thus, $E(V)$ = 0.667, $E(TB)$ = 0.467 and $E_0(V)$ = 0.542. Using Equation (20), we obtain $E_2(T)$ = 1.2188 min. Likewise, using Equation (11), $E_1(T)$ = 1.4923 min.

Hence, from a travel time standpoint, the second configuration performs 18.33% better than the first configuration. This result was anticipated, because elevating the output station will save some travel time in the vertical direction. Before selecting a configuration involving an elevated output station, however, the costs associated with such a design should be considered.

Input and Output at the Same Elevation, but at a Midpoint in the Aisle

The third configuration alternative considered is based on the I/O point being located at the center of the rack. Such a configuration can be visualized as having the delivery and take-away conveyors running halfway into the aisle, through a set of rack openings located at the midlevel on either side of the aisle. It is further assumed that vertical travel time is between 0 and b, and horizontal travel time is between 0 and 1. Hence, the I/O point is assumed to be located at (1/2, b/2) for the normalized rack where $0 < b < 1$.

Let the expected travel time from the center of the rack to a randomly selected point be denoted by $E_M(V)$. Using an approach similar to that one used for deriving $E(SC)$, it can be shown that

$$E_M(V) = \frac{1}{12}b^2 + \frac{1}{4}. \qquad (21)$$

Next consider dwell point strategy A described earlier. Since the input and output stations are coincident for the third configuration, the strategy is equivalent to the case where every trip originates and terminates at the I/O point. Hence, the expected travel time per operation, say $E_3(T)$, will be

$$E_3(T) = \alpha[2E_M(V)] + (1 - \alpha)[2E_M(V) + E(TB)]. \qquad (22)$$

It is instructive to briefly compare the results for the second configuration with the results for the third configuration. Due to the central location of the I/O point, it is expected that the third configuration will perform better from a travel time standpoint. However, the magnitude of the improvement should be determined in order to economically justify higher conveyor costs and the loss of the rack openings required for the conveyor. Again for comparison's sake assume $\alpha = 0.50$, $d = 0.50b$, and $b = 1$. Thus, $E(V) = 0.667$, $E(TB) = 0.467$, $E_0(V) = 0.542$, and $E_M(V) = 0.333$. Hence, $E_2(T) = 1.2188$ min and from Equation (22), $E_3(T) = 0.8995$ min. This implies a 26.2% reduction in expected travel time per operation.

Input and Output Elevated at the End of the Aisle

The fourth configuration alternative treated considers the situation where the I/O station has the location $(0,d)$. As before, it is assumed that the maximum horizontal and vertical travel times are 1.0 and b, respectively.

Recall the analysis of the configuration involving input and output stations at the end of the aisle, but at different elevations. From the results obtained, it is straightforward to obtain the following expressions for the expected travel times for single command and dual command cycles:

$$E(SC) = \frac{1}{3}b^2 + 1 - d(b - d)\,. \tag{23}$$

$$E(DC) = \frac{4}{3} + \frac{1}{2}b^2 - \frac{1}{30}b^3 - d\,(b - d). \tag{24}$$

Comparing Equations (23 and 24) with Equations (3 and 10), elevating the I/O station d time units introduces a correction factor of $d(b - d)$ in the computation of cycle times.

ALTERNATIVE STORAGE METHODS

In designing an AS/RS a number of issues must be considered, including the physical configuration of the storage system, the location of the I/O station, the dwell point strategy, and the storage method to be used. In this paper, it was assumed that randomized storage was used, since our experience indicates it is the most commonly used storage method in industry.

As noted in [17], "two storage location methods that in some sense represent extreme points of view are randomized storage and dedicated storage." In comparison to dedicated storage, randomized storage generally requires less storage space since *the maximum of the aggregate storage requirement is generally less than the aggregate of the maximum storage requirements for each item in storage.* In comparison to randomized storage, dedicated storage results in reduced travel times (or greater throughput) *assuming equal storage areas.*

Goetschalckx [5] has shown that the throughput rate for a smaller randomized storage system can be greater than the throughput rate for a larger dedicated storage system. Previous comparisons of throughput rates under deterministic conditions for randomized storage and dedicated storage have assumed constant storage requirements.

The statistical approach used in this paper for determining expected travel times can be extended to include dedicated storage, using the same square-L storage boundaries assumed by Graves, Hausman, and Schwarz. However, the extension is very tedious; it is complicated by the number of combinations of shapes that can occur for the storage regions in a rack that is not square in time.

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