CONVEYOR ANALYSIS:

"closed-loop, irreversible, or re-circulating conveyors with discretly spaced carriers"

- Deterministic model of a towline or trolley conveyor, powered belt and roller conveyors

Kwo @ GENERAL ELECTRIC (deterministic)

Mayers @ WESTERN ELECTRIC (probabilistic with multi-loading stations)

∴ Deterministic model of materials flow on a conveyor having one loading station and one unloading station.

Issues to be worked on:

- Input and output rates,
- Operating problems (in overhead trolley conveyors),
  - at a loading station, no empty carriers were available when a loading operation was to be performed.
  - at an unloading station, no loaded carriers were available when an unloading operation was to be performed.
- Relationships among the design parameters and operating variables

A conveyor consists of: - the conveyor equipment,
- loading & unloading stations,
- operating discipline.

System design parameters include:

- equipment parameters,
- waiting space allowances,
- number, spacing and sequencing of loading and unloading stations.

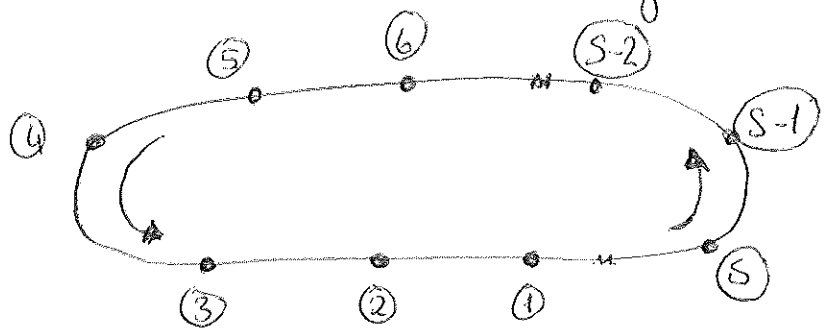
Conveyors are used not only to transport or deliver materials, but also to provide storage capacity.

- some are basically handling or transport,
- some are storage or accumulation,
- some perform both handling and storage.

The general principles in designing closed-loop, irreversible conveyors:

1. Uniformity: uniformly distributed over the conveyor,
2. Capacity: carrying capacity of the conveyor must be greater than or equal to the system throughput requirements.
3. Speed: speed must be within permissible range, defined by loading and unloading station requirements and the technological capabili.

There are equally spaced "k" carriers.
There are "s" loading and/or unloading stations.



Station 1 is used as a reference point.

Carrier n becomes carrier n+k immediately after passing station 1.

$\{t_n\}$: Sequence of points in time at which a carrier passes station 1.

t_n is the time at which carrier n passes station 1.

$f_i(n)$: the amount of material loaded on carrier "n" as it passes station i, $i = 1..s$.

Negative values are denoting unloading.

$H_i(n)$: the amount of material carried by carrier "n" immediately after passing station i.

Long-time behaviour (steady-state operations):

Total amount of material loaded on the conveyor must be equal to the total amount of material unloaded.

The sequences $\{f_i(n)\}$ are assumed to be periodic with period "p".

$$\{f_i(n)\} = \{f_i(n+p)\}$$

$$\sum_{n=1}^{p} \sum_{i=1}^{S} f_i(n) = 0 \qquad \text{Additionally; } F_1(n) = \sum_{i=1}^{S} f_i(n)$$

Results of Muth: From his descriptive model.

- k/p can not be integer for steady-state operations
- let $r = k \bmod p$; r/p must be a proper fraction for general sequences $\{F_1(n)\}$ to be accomodated.
- it is desirable for "p" to be a prime number, as conveyor compatibility results for all admissable values of k.

The material balance equation for carrier n:

$$H_1(n) = H_1(n-r) + F_1(n)$$

⊛ Solution approach in order to determine the values of $H_1(n)$:

Let $H_1^*(n) = 0$ "in order to solve recursively"

$$H_1^{**}(n) = H_1^{*}(n-r) + F_1(n)$$

$$H^*_{i+1}(n) = H^*_i(n) - f_i(n)$$

$$C = \min_{i,n} H^*_i(n)$$

Then; $H_i(n) = H^*_i(n) - C$

Finally; $B = \max_{i,n} H_i(n)$ "required capacity per carrier"

Example: "Deterministic Model"

- Single loading and single unloading stations,
- with equally spaced nine carriers,
- loading and unloading sequences have periods of 7 time units.

$\{f_1(n)\} = \{1, 1, 2, 2, 2, 1, 1\}$

$\{f_2(n)\} = \{0, 0, 0, 0, 0, -5, -5\}$

$\{F_1(n)\} = \{1, 1, 2, 2, 2, -4, -4\}$

$k/p = 9/7$ is not an integer. ✓

$r = k \bmod p$

$r = 9 \bmod 7 = 2$

$r/p = 2/7$ is a proper fraction. ✓

$p = 7$ is a prime number. ✓

Start with letting $H^*_1(1) = 0$

$\Rightarrow H^*_1(n) = H^*_1(n-r) + F_1(n)$

$H_1^*(3) = \cancel{H_1^*(1)}^{\,0} + F_1(3) = 0 + 2 = 2$

$H_1^*(5) = H_1^*(3) + F_1(5) = 2 + 2 = 4$

$H_1^*(7) = H_1^*(5) + F_1(7) = 4 - 4 = 0$

$H_1^*(9) = H_1^*(2) = H_1^*(7) + F_1(2) = 0 + 1 = 1$

$H_1^*(4) = H_1^*(2) + F_1(4) = 1 + 2 = 3$

$H_1^*(6) = H_1^*(4) + F_1(6) = 3 - 4 = -1$

Hence, $\{H_1^*(n)\} = \{0, 1, 2, 3, 4, -1, 0\}$

Since, $H_2^*(n) = H_1^*(n) - f_1(n)$

$= \{0,1,2,3,4,-1,0\} - \{1,1,2,2,2,1,1\}$

$H_2^*(n) = \{-1, 0, 0, 1, 2, -2, -1\}$

Finding $C = \min_{i,n} H_i^*(n) = -2$ *

As a result in order to obtain the final solution, by subtracting C;

$\Rightarrow H_1(n) = \{2,3,4,5,6,1,2\}$ $\quad B = \max_{i,n} H_i(n) = 6$

$H_2(n) = \{1,2,2,3,4,0,1\}$

∴ Therefore, each carrier must have sufficient capacity to accomodate 6 units of product.

The required carrier capacity "B" is affected by the value of the number of carriers, "k".

n	K = 8 $H_1(n)$	K = 8 $H_2(n)$	K = 9 $H_1(n)$	K = 9 $H_2(n)$	K = 10 $H_1(n)$	K = 10 $H_2(n)$	K = 11 $H_1(n)$	K = 11 $H_2(n)$
1	2	1	2	1	5	4	4	3
2	3	2	3	2	2	1	7	6
3	5	3	4	2	5	3	5	3
4	7	5	5	3	7	5	3	1
5	9	7	6	4	4	2	6	4
6	5	4	1	0	1	0	3	2
7	1	0	2	1	3	2	1	1
	B = 9		B = 6		B = 7		B = 7	

K = 12 ⇒ B = 6

K = 13 ⇒ B = 9

Verify?

$\{f_1(n)\} = \{1, 1, 2, 2, 2, 1, 1\}$

$\{f_2(n)\} = \{0, 0, 0, 0, 0, -5, -5\}$

$\{H_1(n)\} = \{2, 3, 4, 5, 6, 1, 2\}$

$\{H_2(n)\} = \{1, 2, 2, 3, 4, 0, 1\}$

→ 5 units are removed from the sixth carrier at station 2,

→ when it arrives at the first station 1 unit is placed on the sixth carrier,

→ when the sixth carrier passes station 1, its steady state load is $A_1(6) = 1$. #

$k = 8$ $\quad k/p = 8/7$ not integer ✓

$r = 8 \bmod 7 = 1$ ✓

$r/p = 1/7$ a proper fraction ✓

$H_1^*(2) = \cancel{H_1^*(1)}^{0} + F_1(2) = 0 + 1 = 1$

$H_1^*(3) = H_1^*(2) + F_1(3) = 1 + 2 = 3$

$H_1^*(4) = H_1^*(3) + F_1(4) = 3 + 2 = 5$

$H_1^*(5) = H_1^*(4) + F_1(5) = 5 + 2 = 7$

$H_1^*(6) = H_1^*(5) + F_1(6) = 7 + (-4) = 3$

$H_1^*(7) = H_1^*(6) + F_1(7) = 3 + (-4) = -1$

$H_1^*(n) = \{0, 1, 3, 5, 7, 3, -1\}$

$H_2^*(n) = \{0, 1, 3, 5, 7, 3, -1\} - \{1, 1, 2, 2, 2, 1, 1\}$

$H_2^*(n) = \{-1, 0, 1, 3, 5, 2, -2\}$ $\quad C = -2$

$H_1(n) = \{2, 3, 5, 7, 9, 5, 1\}$

$H_2(n) = \{1, 2, 3, 5, 7, 4, 0\}$ $\quad B = 9$

$K = 10$ $\quad K/p = 10/7$ not integer ✓

$r = 10 \bmod 7 = 3$ ✓

$r/p = 3/7$ is a proper fraction ✓

$H_1^*(4) = \cancel{H_1^*(1)}^{0} + F_1(4) = 0 + 2 = 2$

$H_1^*(7) = H_1^*(4) + F_1(7) = 2 + (-4) = -2$

$H_1^*(10) = H_1^*(3) = H_1^*(7) + F_1(3) = -2 + 2 = 0$

$H_1^*(6) = H_1^*(3) + F_1(6) = 0 + (-4) = -4$

$H_1^*(9) = H_1^*(2) = H_1^*(6) + F_1(2) = -4 + 1 = -3$

$H_1^*(12) = H_1^*(5) = H_1^*(2) + F_1(5) = -3 + 2 = -1$

$H_1^*(n) = \{0, -3, 0, 2, -1, -4, -2\}$

$H_2^*(n) = \{0, -3, 0, 2, -1, -4, -2\} - \{1, 1, 2, 2, 2, 1, 1\}$

$H_2^*(n) = \{-1, -4, -2, 0, -3, -5, -3\}$ $\quad c = -5$

$H_1(n) = \{5, 2, 5, 7, 4, 1, 3\}$

$B = 7$

$H_2(n) = \{4, 1, 3, 5, 2, 0, 2\}$

K=11

$H_1(n)$	$H_2(n)$
4	3
7	6
5	3
3	1
6	4
3	2
1	0

B=7

K=12

$H_1(n)$	$H_2(n)$
6	5
5	4
5	3
4	2
3	1
2	1
1	0

B=6

K=13

$H_1(n)$	$H_2(n)$
9	8
8	7
7	5
5	3
3	1
1	0
5	4

B=9

- When the 15th carrier reaches station 2;

$f_2(15) = f_2(8) = f_2(1) = 0$, since $p = 7$,

nothing happens,

But, at station 1;

$f_1(15) = f_1(8) = f_1(1) = 1$, 1 product is to be added to the carrier.

and the total is raised to 2, note that, it is the steady-state value, $H_1(1) = 2$. #

- When the 24th carrier reaches station 2;

$f_2(24) = f_2(17) = f_2(10) = f_2(3) = 0$, nothing happens,

But, at station 1, $f_1(3) = 2$, the content of the carrier reaches 4.

- When the 33rd carrier reaches station 2;

$f_2(33) = f_2(26) = f_2(19) = f_2(12) = f_2(5) = 0$, nothing happens,

But, at station 1, $f_1(5) = 2$, the carrier content is increased to 6 units.

- 42nd carries; $f_2(42) = f_2(7) = -5$, the carrier content is reduced to 1 unit.

$f_1(7)=1$, the content of the carrier is increased to 2 units.

- 51st carrier; $f_2(51)=f_2(2)=0$, nothing happens, $f_1(2)=1$, the content of the carrier is increased to 3 units.
- 60th carrier; $f_2(60)=f_2(4)=0$, nothing happens, $f_1(4)=2$, 2 units will be added, and the content will reach to 5 units.
- 69th carrier; $f_2(69)=f_2(6)=-5$, 5 units will be removed from the carrier.

So, the carrier arrives at station 1, as empty. This is the same with carrier 6, so the cycle repeats.

Content of the carrier as it left station i, followed a sequence which is a permutation of $\{H_i(n)\}$.

Example: A conveyor with 1 loading and 1 unloading stations.

$\{f_1(n)\}=\{0,0,2,3,1\}$ Assuming 7 carriers on

$\{f_2(n)\}=\{0,0,0,-2,-4\}$ the conveyor, solve for

$\{F_1(n)\}=\{0,0,2,1,-3\}$ $\{H_i(n)\}$.

$k/p = 7/5$ not integer ✓

$r = 7 \bmod 5 = 2$ ✓

$2/5$ is a proper fraction ✓

$H_1^*(3) = \cancel{H_1^*(1)}^{0} + F_1(3) = 0 + 2 = 2$

$H_1^*(5) = H_1^*(3) + F_1(5) = 2 + (-3) = -1$

$H_1^*(7) = H_1^*(2) = H_1^*(5) + F_1(2) = -1 + 0 = -1$

$H_1^*(9) = H_1^*(4) = H_1^*(2) + F_1(4) = -1 + 1 = 0$

$H_1^*(n) = \{0, -1, 2, 0, -1\}$

$H_2^*(n) = \{0, -1, 2, 0, -1\} - \{0, 0, 2, 3, 1\} = \{0, -1, 0, -3, -2\}$

$C = -3$

$H_1(n) = \{3, 2, 5, 3, 2\}$
$H_2(n) = \{3, 2, 3, 0, 1\}$
} $\underline{B = 5}$ carrier capacity should be at least 5 units.

Suppose, the distance from station 2 to station 1 on the conveyor is changed.

$\{f_1(n)\} = \{2, 3, 1, 0, 0\}$
$\{f_2(n)\} = \{0, 0, 0, -2, -4\}$.
} Assuming again 7 carriers solve for $\{H_i(n)\}$.

$H_1^*(3) = \cancel{H_1^*(1)}^{0} + F_1(3) = 0 + 1 = 1$

where $\{F_1(n)\} = \{2, 3, 1, -2, -4\}$

$H_1^*(5) = H_1^*(3) + F_1(5) = 1 + (-4) = -3$

$H_1^*(7) = H_1^*(2) = H_1^*(5) + F_1(2) = -3 + 3 = 0$

$H_1^*(9) = H_1^*(4) = H_1^*(2) + F_1(4) = 0 + (-2) = -2$

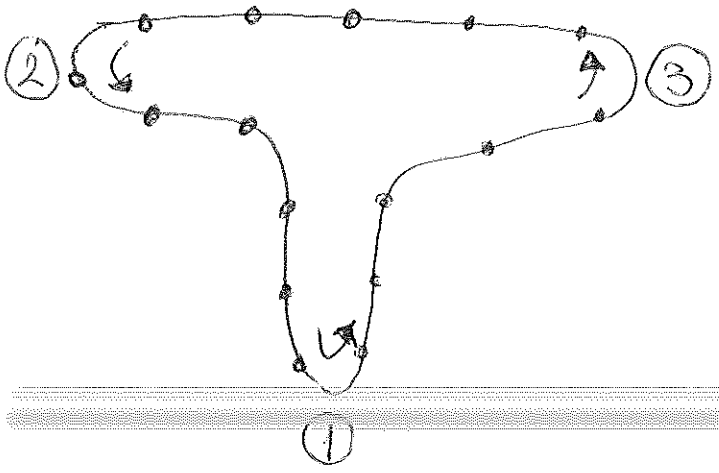
$H_1^*(n) = \{0, 0, 1, -2, -3\}$

$H_2^*(n) = \{0, 0, 1, -2, -3\} - \{2, 3, 1, 0, 0\} = \{-2, -3, 0, -2, -3\}$

$C = -3$

$H_1(n) = \{3, 3, 4, 1, 0\}$
$H_2(n) = \{1, 0, 3, 1, 0\}$ } $B = 4$ "minimum carrier capacity is decreased by 1 units."

Example: A closed loop, irreversible conveyor with oliscretely spaced carriers to serve for three workstations.



$f_1(n) = (4, 4, 0)$ $k = 16$
$f_2(n) = (-3, -1, 0)$ $r = 1$
$f_3(n) = (0, -2, -2)$
$F_1(n) = (1, 1, -2)$

$H_1^*(2) = \cancel{H_1^*(1)}^{0} + F_1(2) = 1$

$H_1^*(3) = H_1^*(2) + F_1(3) = 1 + (-2) = -1$

$H_1^*(n) = (0, 1, -1)$

$H_2^*(n) = (0, 1, -1) - (4, 4, 0) = (-4, -3, -1)$

$H_3^*(n) = (-4, -3, -1) - (-3, -1, 0) = (-1, -2, -1)$

$C = -4$

$H_1(n) = (4, 5, 3)$
$H_2(n) = (0, 1, 3)$
$H_3(n) = (3, 2, 3)$

$B = 5$ *

If workstations will be re-located as;

$f_1(n) = (0, -2, -2)$
$f_2(n) = (4, 4, 0)$
$f_3(n) = (-3, -1, 0)$
$F_1(n) = (1, 1, -2)$

$H_1^*(2) = \cancel{H_1^*(1)}^{0} + F_1(2) = 1$

$H_1^*(3) = 1 + F_1(3) = -1$

$H_1^*(n) = (0, 1, -1)$

$H_2^*(n) = (0, 1, -1) - (0, -2, -2) = (0, 3, 1)$

$H_3^*(n) = (0, 3, 1) - (4, 4, 0) = (-4, -1, 1)$

$C = -4$

$H_1(n) = (4, 5, 3)$
$H_2(n) = (4, 7, 5)$
$H_3(n) = (0, 3, 5)$

$B = 7$ *

∴ The locations of workstations affect the analysis of the closed-loop, recirculating conveyor, by changing $\Sigma H_i(n)$'s.

Alternative locations of workstations,

+

Alternative conveyor lengths, measured by # of carriers,

+

Economic performance and the cost model,

⇒ OPTIMUM CONVEYOR DESIGN!!!

HORSEPOWER CACCULATIONS in CONVEYOR DESIGN:

The horsepower requirement depends on the conveyor speed, and the weight of the conveyor and the material it transports.

An approximate model for powered belt and roller conveyors. Our purpose is not to be an expert in determining horsepower requirements. But, being aware of which components affect the requirements.

HP = horse power requirement (hp)

S = speed of the conveyor (fpm, feet per minute)

L = load to be carried (lb, pounds)

TL = total length of the conveyor (feet)

RC = spacing between centerlines of rollers (inch)

WBR = width between rails (inch)
(it should be at least 2.5" longer than the width of the load being transported.)

α = angle of incline, (°, degree)

LL_1 = live load on incline (lb, pounds)

BV = base value $\quad BV \approx \frac{2}{3} * WBR$

FF = friction factor $\quad FF = 0.05$ (for roller supported)
$= 0.30$ (for slider bed supported)

LF = length factor
(tabulated LF value for belt conveyors, based on given RC & WBR values.)

For belt conveyors:

$$HP = \frac{[BV + LF(TL) + FF(L) + LL_1(\sin\alpha)] \cdot S}{14{,}000}$$

Example = • 100ft long roller supported belt conveyor,
• angle of 10° (for a maximum of 35° incline),
• with a 6 inches spacing,
• WBR of 27 inches,

- each totebox is 18 inches and weighing 35 lb (loaded)

(for stability, at least two rollers be under the load, a spacing of 12 inches to be provided between toteboxes.)

- Conveyor speed is to be 90ft/min,
- load segment is 30 inches, exactly 40 load segments will be on the 100ft-long conveyor.
- Weight of the load will be $40 \cdot 35 = 1400$ lb

$S = 90$ fpm

$RC = 6''$

$WBR = 27''$

$BV = \frac{2}{3} * 27 = 18''$

$LF = 0.61$

$TL = 100'$

$FF = 0.05$

$L = LL_1 = 1400$ lb

$\alpha = 10°$

$$HP = \frac{[18 + 0.61(100) + 0.05(1400) + 1400(0.1737)] * 90}{14.000}$$

$HP = 2.52$ hp * ∴ 2.75 or 3.00hp should meet the demands placed on the belt conveyor.

⊛ What if spacing between toteboxes had been 8 inches, rather than 12 inches.

$\frac{100 * 12}{26} = 46.15$ (46 complete load segments + a partial load segment)

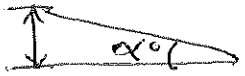
$46 \times 26 = 1196''$ ⇒ 46 full toteboxes and a 4' of a partial totebox.

$\frac{4}{18} * 35 = 7.78 lb$

Maximum load $46 * 35 + 7.78 = 1,617.78 lb.$

$$HP = \frac{\left[18 + 0.61(100) + 0.05(1,617.78) + 1,617.78 * (0.1737)\right] * 90}{14,000}$$

$HP = \underline{2.83 hp}$ *

⊛ Suppose it is desired to elevate 17.36' 

Alternatives:	10°	20°	30°
Conveyor lengths:	100ft	51ft	35ft
HP requirements:	2.52hp.	?hp	?hp

→ $\alpha = 20°$, TL = 51ft, 20 complete load segments + 12 inches

Thus, $L = 20(35) + \left(\frac{12}{18}\right) * 35 = \underline{723.33 lb}$

$$HP = \frac{[18 + 0.61(51) + 0.05(723.33) + 723.33(\sin 20°)] * 90}{14{,}000}$$

(51 → 31.11; 723.33 → 36.167; $\sin 20° \rightarrow 0.342$; $723.33 \times 0.342 \rightarrow 247.39$)

(17)

$HP = \underline{2.14\ hp}$ ✱

➡ $\alpha = 30°$, $TL = 35ft$, 14 complete load segments

Thus, $L = LL_1 = 14 \cdot 35 = \underline{490\ ft}$

$$HP = \frac{[18 + 0.61(35) + 0.05(490) + 490(\sin 30°)] * 90}{14{,}000}$$

(35 → 21.35; 490 → 24.50; $\sin 30° \rightarrow 0.5$; $490 \times 0.5 \rightarrow 245$)

$= \underline{1.99\ hp}$ ✱

∴ The steepest feasible angle of incline would be preferred since it requires a smaller horsepower motor and a shorter conveyor.

⊛ Suppose 100ft horizontal distance, if 2.5hp motor is available and must be used, what could be the maximum weight per totebox?

$S = 90fpm$ $\quad TL = 100'$

$RC = 6''$ $\quad FF = 0.04$

$WBR = 27''$ $\quad \alpha = 0°$

$BV = (\frac{2}{3})27 = 18''$

$LF = 0.61$

$$HP = \frac{[18 + 0.61(100) + 0.05L] * 90}{14{,}000} = 2.5$$

✱

$$L = \left[\frac{2.5 * 14{,}000}{90} - 18 - 0.61(100)\right] \cdot \frac{1}{0.05} = \underline{6.197.78\ lb}$$

(388.89 and 61)

Since there will be 40 toteboxes, the maximum weight per totebox is $\frac{6.197.78}{40} = \underline{154.94\ lb}$ #

(*) For roller conveyors;

FF = 0.10 a flat belt

= 0.085 zero pressure accumulating belt

= 0.075 V-belt

= 0.05 chain driven rollers

BV = 4.60 + 0.445(WBR)

Tabulated LF values for various RC and WBR values.

For an 18 inches long load, the largest roller spacing is 6 inches.

Example: S = 90fpm, RC = 6 inches, WBR = 27 inches

BV = 4.60 + 0.445(27) = 16.62", LF = 1.0

TL = 100ft, FF = 0.10, L = 1400 lb.

(Note that, LL_1 = 0 lb, since $\alpha = 0°$)

$$HP = \frac{[16.62 + 1.0(100) + 0.1(1400)] * 90}{14.000} = \underline{1.6497\ hp}$$ #

∴ 1.75 or 2.00 hp is an appropriate motor selection.

(*) If the motor is 2.5hp, than what should be the maximum weight of the totebox?

$$HP = \frac{[16.62 + 1.0(100) + 0.10(L)] * 90}{14.000} = 2.5\,hp$$

$$L = \left[\frac{2.5 * 14.000}{90} - 16.62 - 100\right] \cdot \frac{1}{0.10} = \underline{2.722.69\,lb}$$

$$\text{Maximum weight per totebox} = \frac{2.722.69}{40} = \underline{68.07\,lb}\ *$$

Example:

18 inches toteboxes move with 90fpm with a clearance of 12 inches.

$$90\,ft/min * (12\,in/ft) / (30\,in/totebox) = 36\ toteboxes/min$$

This is the rate entering to the second conveyor.

$$\text{Spacing in the second conveyor} = (150\,ft/min)(12\,in/feet) / (36\ toteboxes/min)$$

$$= 50\ inches/totebox$$

∴ Since, toteboxes are 18 inches, the clearence will be 32 inches in the second conveyor.

Increase in speed is not linear with the increase in spacing.

$\frac{100 * 12}{50} = 24$ complete load segments will be in the second conveyor.

S = 150 fpm, RC = 6 inches, WBR = 27 inches

BV = 16.62 inches, LF = 1.0, TL = 100 ft, FF = 0.10

L = 24 * 35 = 840 lb

$$HP = \frac{[16.62 + 1.0(100) + 0.10(840)] * 150}{14.000} = \underline{2.1495 \text{ hp}}_{*}$$

(*) 90 fpm, 100 ft → 1.65 hp; 150 fpm, 100 ft → 2.15 hp; 120 fpm, 200 ft → ? hp; 36 totebaxes/min

Length of one load segment is 40 inches,

60 complete load segments.

S = 120 fpm, RC = 6 inches, WBR = 27 inches

BV = 16.62, LF = 1.0, TL = 200 ft, FF = 0.10

L = 60 * 35 = 2.100 lb

$$HP = \frac{[16.62 + 1.0(200) + 0.10(2.100)] * 120}{14.000} = \underline{3.66 \text{ hp}}_{**}$$

∴ Alternatives could be compared and cost optimization could be possible.